

Exercise 4

The point $P(0.5, 0)$ lies on the curve $y = \cos \pi x$.

- (a) If Q is the point $(x, \cos \pi x)$, use your calculator to find the slope of the secant line PQ (correct to six decimal places) for the following values of x :

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|------------|--------------|------------|
| (i) 0 | (ii) 0.4 | (iii) 0.49 |
| (iv) 0.499 | (v) 1 | (vi) 0.6 |
| (vii) 0.51 | (viii) 0.501 | |

- (b) Using the results of part (a), guess the value of the slope of the tangent line to the curve at $P(0.5, 0)$.
- (c) Using the slope from part (b), find an equation of the tangent line to the curve at $P(0.5, 0)$.
- (d) Sketch the curve, two of the secant lines, and the tangent line.

Solution

For each value of x , calculate y and then use the slope formula to find the slope of PQ .

$x = 0$	$\rightarrow y = 1 :$	$Q(0, 1)$	$\Rightarrow m = \frac{1 - 0}{0 - 0.5} = -2$
$x = 0.4$	$\rightarrow y \approx 0.309017 :$	$Q(0.4, 0.309017)$	$\Rightarrow m = \frac{0.309017 - 0}{0.4 - 0.5} = -3.09017$
$x = 0.49$	$\rightarrow y \approx 0.031411 :$	$Q(0.49, 0.031411)$	$\Rightarrow m = \frac{0.031411 - 0}{0.49 - 0.5} = -3.1411$
$x = 0.499$	$\rightarrow y \approx 0.003142 :$	$Q(0.499, 0.003142)$	$\Rightarrow m = \frac{0.003142 - 0}{0.499 - 0.5} = -3.142$
$x = 0.501$	$\rightarrow y \approx -0.003142 :$	$Q(0.501, -0.003142)$	$\Rightarrow m = \frac{-0.003142 - 0}{0.501 - 0.5} = -3.142$
$x = 0.51$	$\rightarrow y \approx -0.031411 :$	$Q(0.51, -0.031411)$	$\Rightarrow m = \frac{-0.031411 - 0}{0.51 - 0.5} = -3.1411$
$x = 0.6$	$\rightarrow y \approx -0.309017 :$	$Q(0.6, -0.309017)$	$\Rightarrow m = \frac{-0.309017 - 0}{0.6 - 0.5} = -3.09017$
$x = 1$	$\rightarrow y = -1 :$	$Q(1, -1)$	$\Rightarrow m = \frac{-1 - 0}{1 - 0.5} = -2$

The slope of the tangent line at $x = 0.5$ is presumably $-\pi$. Therefore, the equation of the tangent line is

$$y - 0 = -\pi(x - 0.5),$$

or

$$y = -\pi x + \frac{\pi}{2}.$$

Note that the secant line going through $(0, 1)$ and $(0.5, 0)$ is the same as the secant line going through $(0.5, 0)$ and $(1, -1)$.

$$y - 1 = -2(x - 0)$$

$$y = 1 - 2x$$

$$y + 1 = -2(x - 1)$$

$$y = 1 - 2x$$

